

6. Zjednodušte a uveďte, kdy má výraz smysl. Určete jeho hodnotu pro  $a = -2, b = 1$ .

$$\left(\frac{a}{b}\right)^2 \cdot \frac{2a^2}{b^4} = \frac{a^2}{b^2} \cdot \frac{2a^2}{b^4} = \frac{2a^4}{b^6} \quad \boxed{b \neq 0}$$

$$\frac{(a-1)^2}{1-b} \cdot \frac{b-1}{1+a} = \frac{(a-1)(a-1)}{\cancel{1-b}} \cdot \frac{(-1)\cancel{(1-b)}}{1+a} = \frac{(-1)(a-1)(a-1)}{1+a} \quad \boxed{\begin{array}{l} b \neq 1 \\ a \neq -1 \end{array}}$$

$$\left(\frac{a^2}{-b}\right)^2 \cdot \left(-\frac{2b}{4a}\right)^3 = \frac{a^4}{b^2} \cdot \left(-\frac{1}{2a}\right)^3 = -\frac{ab}{8} \quad \boxed{\begin{array}{l} a \neq 0 \\ b \neq 0 \end{array}}$$

$$\frac{4a^2+8a+4}{ab^2-b} \cdot \frac{1-ab}{4a+4} = \frac{4 \cdot (a^2+2a+1)}{b(ab-1)} \cdot \frac{1-ab}{4(a+1)} = \frac{\cancel{4} \cdot (a+1)^2}{b \cdot \cancel{(ab-1)}} \cdot \frac{(-1)\cancel{(ab-1)}}{\cancel{4}(a+1)} = \frac{(-1)(a+1)}{b}$$

$\boxed{\begin{array}{l} b \neq 0 \\ a \neq -1 \end{array}} \rightarrow \begin{array}{l} ab-1=0 \\ ab=1 : : b \\ a = \frac{1}{b} \end{array}$

7. Zjednodušte výrazy a výsledky ověřte pro dané hodnoty proměnných.  $a = -1, b = 2, c = 1$ .

$$\frac{ab+b^2-3a}{9} \cdot \frac{-3a}{b^2} = \frac{\cancel{b}(a+b)}{9_3} \cdot \frac{-\cancel{3}a}{b^2} = \frac{-a(a+b)}{3b} \quad \boxed{b \neq 0}$$

$$\frac{a-ab}{c-ac} \cdot \frac{ac-c}{ab-a} = \frac{\cancel{a}(1-b)}{\cancel{c}(1-a)} \cdot \frac{\cancel{c}(a-1)}{\cancel{a}(b-a)} = \frac{\cancel{a}}{\cancel{c}} \cdot \frac{(-1)\cancel{(1-a)}}{(-1)\cancel{(1-a)}} = 1 \quad \boxed{\begin{array}{l} a \neq 1 \\ b \neq 1 \\ a \neq 0 \\ c \neq 0 \end{array}}$$

$$\frac{a^2-ab}{b} \cdot \frac{ab+b^2}{a} = \frac{\cancel{a}(a-b)}{\cancel{b}} \cdot \frac{\cancel{b}(a+b)}{\cancel{a}} = (a-b)(a+b) = a^2 - b^2 \quad \boxed{\begin{array}{l} a \neq 0 \\ b \neq 0 \end{array}}$$

$$\frac{a^3-a^2}{b^3-ab^2} \cdot \frac{a-b}{a-a^2} = \frac{\cancel{a^2}(a-1)}{b^2(b-a)} \cdot \frac{a-b}{\cancel{a}(1-a)} = \frac{\cancel{a}(a-1)}{b^2\cancel{(1-a)}} \cdot \frac{\cancel{(1-a)}}{\cancel{(1)}(a-1)} = \frac{a}{b^2} \quad \boxed{\begin{array}{l} a \neq 0 \\ b \neq 0 \\ a \neq 1 \end{array}}$$

1. Zjednodušte výrazy.

$$\frac{p}{q^3} : \frac{p^2}{q} = \frac{\cancel{p}}{q^{\cancel{3}_2}} \cdot \frac{\cancel{q}}{\cancel{p}} = \frac{1}{q^2} \quad \begin{matrix} p \neq 0 \\ q \neq 0 \end{matrix}$$

$$\frac{2a}{3b} : \frac{4a}{6b^2} = \frac{\cancel{2}a}{\cancel{3}b} \cdot \frac{\cancel{b}^2}{\cancel{4}b} = \frac{2b}{1} = 2b \quad \begin{matrix} a \neq 0 \\ b \neq 0 \end{matrix}$$

$$\frac{-6a^2b}{3c} : \frac{3ab}{2c^2} = \frac{\cancel{-6}a^{\cancel{2}_1}b}{\cancel{3}c} \cdot \frac{\cancel{2}c^{\cancel{2}_1}}{\cancel{3}ab} = \frac{-4ac}{3} \quad \begin{matrix} a \neq 0 \\ b \neq 0 \\ c \neq 0 \end{matrix}$$

$$\frac{k^2l}{3m} : \frac{6k}{m^2} = \frac{\cancel{k}^2l}{\cancel{3}m} \cdot \frac{\cancel{m}^2}{\cancel{6}k} = \frac{k^2lm}{1k} = klm \quad \begin{matrix} k \neq 0 \\ m \neq 0 \end{matrix}$$

$$\frac{1}{x^2y} : \frac{2}{xy} = \frac{1}{x^{\cancel{2}_1}y} \cdot \frac{\cancel{xy}}{2} = \frac{1}{2x} \quad \begin{matrix} x \neq 0 \\ y \neq 0 \end{matrix}$$

$$\frac{-2u^2}{4v^2} : \frac{-u}{v} = \frac{\cancel{-2}u^{\cancel{2}_1}}{\cancel{4}v^{\cancel{2}_1}} \cdot \frac{\cancel{v}}{\cancel{-u}} = \frac{u}{2v} \quad \begin{matrix} u \neq 0 \\ v \neq 0 \end{matrix}$$

$$4ab : \frac{a^2b}{3} = \frac{\cancel{4}a^{\cancel{1}_0}b}{1} \cdot \frac{3}{\cancel{a}^{\cancel{2}_1}b} = \frac{12}{a} \quad \begin{matrix} a \neq 0 \\ b \neq 0 \end{matrix}$$

$$\frac{r^2s}{7s^2} : (-rs) = \frac{\cancel{r}^{\cancel{2}_1}s}{\cancel{7}s^{\cancel{2}_1}} \cdot \frac{1}{\cancel{-r}s} = \frac{r}{7s^2} \quad \begin{matrix} r \neq 0 \\ s \neq 0 \end{matrix}$$

2. Vydělte a určete, kdy má výraz smysl.

$$\frac{21v^3}{4u^2} : 7v = \frac{\cancel{21}v^{\cancel{3}_2}}{\cancel{4}u^{\cancel{2}_1}} \cdot \frac{1}{\cancel{7}v} = \frac{3v^2}{4u^2} \quad \begin{matrix} u \neq 0 \\ v \neq 0 \end{matrix}$$

$$\frac{k^2+k}{k+1} : k^2 = \frac{\cancel{k}^{\cancel{2}_1}(\cancel{k}+1)}{\cancel{k}+1} \cdot \frac{1}{k^{\cancel{2}_1}} = \frac{1}{k} \quad \begin{matrix} k \neq 0 \\ k \neq -1 \end{matrix}$$

$$\frac{4xy}{3ab} : 10x^2y = \frac{\cancel{4}x^{\cancel{1}_0}y}{\cancel{3}ab} \cdot \frac{1}{\cancel{10}x^{\cancel{2}_1}y} = \frac{2}{15ab} \quad \begin{matrix} a \neq 0 & x \neq 0 \\ b \neq 0 & y \neq 0 \end{matrix}$$

$$14m^2n^2 : \frac{10m^3}{3n} = \frac{\cancel{14}m^{\cancel{2}_1}n^{\cancel{2}_1}}{1} \cdot \frac{\cancel{3}n}{\cancel{10}m^{\cancel{3}_2}} = \frac{21m^2n}{2m} \quad \begin{matrix} m \neq 0 \\ n \neq 0 \end{matrix}$$

$$\frac{3(x+1)}{x^2y} : \frac{2x+2}{3} = \frac{\cancel{3}(x+1)}{x^{\cancel{2}_1}y} \cdot \frac{3}{\cancel{2}(x+1)} = \frac{9}{2x^2y} \quad \begin{matrix} x \neq 0 \\ y \neq 0 \\ x \neq -1 \end{matrix}$$

$$\frac{r+s}{r} : \frac{r^2+rs}{4r} = \frac{\cancel{r+s}}{r} \cdot \frac{\cancel{4}r}{r(\cancel{r+s})} = \frac{4}{r} \quad \begin{matrix} r \neq 0 \\ r \neq -s \end{matrix}$$

$$(a-b) : \frac{2a-2b}{3a} = \frac{\cancel{a-b}}{1} \cdot \frac{\cancel{3}a}{\cancel{2}(a-b)} = \frac{3a}{2} \quad \begin{matrix} a \neq 0 \\ a \neq b \end{matrix}$$

$$(2x+8) : \frac{x+4}{x^2-xy} = \frac{\cancel{2}(x+4)}{1} \cdot \frac{x(x-y)}{\cancel{x+4}} = \frac{2x(x-y)}{1} \quad \begin{matrix} x \neq 0 \\ x \neq y \\ x \neq -4 \end{matrix}$$

$$\frac{t-2}{1-t^2} : \frac{3t-6}{1+t} = \frac{\cancel{t-2}}{(1-t)(\cancel{1+t})} \cdot \frac{\cancel{1+t}}{\cancel{3}(t-2)} = \frac{1}{3(1-t)} \quad \begin{matrix} t \neq 1 \\ t \neq -1 \\ t \neq 2 \end{matrix}$$

3. Vydělte a určete, kdy má výraz smysl.

$$\frac{a^2 - b^2}{a + b} : \frac{a - b}{a} = \frac{\cancel{(a+b)} \cancel{(a+b)}}{\cancel{a+b}} \cdot \frac{a}{\cancel{a+b}} = \underline{a} \quad \begin{cases} a \neq -b \\ a \neq 0 \end{cases}$$

$$\frac{(r+1)^2}{r-1} : \frac{r+1}{(r-1)^2} = \frac{\cancel{(r+1)} \cancel{(r+1)}}{\cancel{r-1}} \cdot \frac{\cancel{(r-1)} \cancel{(r-1)}}{\cancel{r-1}} = (r+1)(r-1) = \underline{r^2 - 1} \quad \begin{cases} r \neq 1 \\ r \neq -1 \end{cases}$$

$$\frac{a^2 + ab}{ab + b^2} : \frac{1}{b^2} = \frac{a \cancel{(a+b)}}{\cancel{b} \cancel{(a+b)}} \cdot \frac{b^2}{1} = \underline{ab} \quad \begin{cases} b \neq 0 \\ a \neq -b \end{cases}$$

$$\frac{x-2}{x^2+2x+1} : \frac{2x-4}{x+1} = \frac{\cancel{x-2}}{(x+1)^2} \cdot \frac{\cancel{x+1}}{2 \cancel{(x-2)}} = \frac{1}{2(x+1)} \quad \begin{cases} x \neq -1 \\ x \neq 2 \end{cases}$$

$$\frac{r^2 - 1}{r^2 + 2r + 1} : \frac{1 - r}{3r + 3} = \frac{\cancel{(r+1)} \cancel{(r-1)}}{(\cancel{r+1})^2} \cdot \frac{3 \cancel{(r+1)}}{1 - r} = \frac{r-1}{1-r} = \frac{\cancel{r-1}}{(-1) \cancel{(r-1)}} = \underline{-1} \quad \begin{cases} r \neq 1 \\ r \neq -1 \end{cases}$$

4. Zjednodušte a určete, kdy má výraz smysl.

$$\frac{a^2 - n^2}{(a+n)^2} : \frac{n-a}{4a+4n} = \frac{\cancel{(a+n)} \cancel{(a-n)}}{(\cancel{a+n})^2} \cdot \frac{4 \cancel{(a+n)}}{m-a} = \frac{4 \cdot (a-n)}{(n-a)} = \frac{4 \cancel{(a-n)}}{(-1) \cancel{(a-n)}} = \underline{-4} \quad \begin{cases} a \neq -n \\ a \neq n \end{cases}$$

$$\frac{-2xz}{21uv^2} : \frac{-uvx}{14u^2vx} = \frac{-2xz}{\cancel{2} \cancel{u} \cancel{v}^2} \cdot \frac{\cancel{14} \cancel{u}^2 \cancel{v} \cancel{x}}{-\cancel{u} \cancel{v} \cancel{x}} = \frac{4xz}{3u^2} \quad \begin{cases} u \neq 0 \\ v \neq 0 \\ x \neq 0 \end{cases}$$

$$\frac{15+15n}{n^2-1} : \frac{5}{n^3-n} = \frac{3 \cdot 5 \cdot (1+n)}{(n+1)(n-1)} \cdot \frac{n \cdot (n^2-1)}{5} = \frac{3(1+n) \cdot n \cdot \cancel{(n+1)} \cancel{(n-1)}}{(n+1)(n-1)} = \underline{3n(1+n)} \quad \begin{cases} n \neq 1 \\ n \neq -1 \\ n \neq 0 \end{cases}$$

$$\frac{a^2 - 25}{a^2 + 10a + 25} : \frac{10 - 2a}{a^2 + 5a} = \frac{(a-5) \cancel{(a+5)}}{(\cancel{a+5})^2} \cdot \frac{a \cancel{(a+5)}}{2(5-a)} = \frac{a \cancel{(a-5)}}{2 \cdot (-1) \cdot \cancel{(a-5)}} = \underline{-\frac{a}{2}} \quad \begin{cases} a \neq -5 \\ a \neq 5 \end{cases}$$

$$\frac{9x^2 - 18xy + 9y^2}{x^2y - xy^2} : \frac{3x-3y}{x^2y} = \frac{9 \cdot (x^2 - 2xy + y^2)}{xy(x-y)} \cdot \frac{x^2y}{3(x-y)} = \frac{3 \cdot 9 \cdot \cancel{(x-y)}^2}{\cancel{xy} \cancel{(x-y)}} \cdot \frac{\cancel{x} \cancel{y}}{3 \cancel{(x-y)}} = \frac{3x}{1} = \underline{3x} \quad \begin{cases} x \neq 0 \\ y \neq 0 \\ x \neq y \end{cases}$$



1. Upravte, určete podmínky platnosti.

$$\frac{\frac{4}{a}}{\frac{2}{ab}} = \frac{4}{a} : \frac{2}{ab} = \frac{4}{a} \cdot \frac{ab}{2} = 2b \quad \begin{matrix} a \neq 0 \\ b \neq 0 \end{matrix}$$

$$\frac{\frac{5x}{3y^3}}{\frac{10x^3}{3y}} = \frac{5x}{3y^3} : \frac{10x^3}{3y} = \frac{5x}{3y^3} \cdot \frac{3y}{10x^3} = \frac{1}{2x^2y^2} \quad \begin{matrix} x \neq 0 \\ y \neq 0 \end{matrix}$$

$$\frac{\frac{r}{s^2}}{\frac{8r^4}{15s^5}} = \frac{r}{s^2} : \frac{8r^4}{15s^5} = \frac{r}{s^2} \cdot \frac{15s^5}{8r^4} = \frac{15s^3}{8r^3} \quad \begin{matrix} r \neq 0 \\ s \neq 0 \end{matrix}$$

$$\frac{\frac{5z^2}{2z^3}}{5x} = \frac{5z^2}{2z^3} : \frac{5x}{1} = \frac{5z^2}{2z^3} \cdot \frac{1}{5x} = \frac{z}{2xz} = \frac{1}{2x} \quad \begin{matrix} x \neq 0 \\ z \neq 0 \end{matrix}$$

$$\frac{\frac{2a-6}{3b}}{\frac{ab-3b}{12b}} = \frac{2(a-3)}{3b} : \frac{b(a-3)}{12b} = \frac{2(a-3)}{3b} \cdot \frac{12b}{b(a-3)} = \frac{8}{1} \quad \begin{matrix} b \neq 0 \\ a \neq 3 \end{matrix}$$

$$\frac{\frac{6a^2-24}{1-a}}{\frac{8a-16}{a-1}} = \frac{6(a^2-4)}{1-a} : \frac{8(a-2)}{a-1} = \frac{6(a-2)(a+2)}{1-a} \cdot \frac{(a-1)}{8(a-2)} = \frac{-3(a+2)}{4} \quad \begin{matrix} a \neq 1 \\ a \neq 2 \end{matrix}$$

$$\frac{\frac{x-1}{y}}{\frac{2x-2y}{y^2}} = \frac{x-1}{y} : \frac{2(x-1)}{y^2} = \frac{x-1}{y} \cdot \frac{y^2}{2(x-1)} = \frac{y}{2} \quad \begin{matrix} y \neq 0 \\ x \neq 1 \end{matrix}$$

$$\frac{\frac{2-\frac{2}{x}}{\frac{1}{1-x}}}{x} = \frac{2x-2}{x} : \frac{1-x}{x} = \frac{2(x-1)}{x} \cdot \frac{x}{1-x} = \frac{2(x-1)}{1-x} = \frac{2(x-1)}{-(1-x)} = -2 \quad \begin{matrix} x \neq 0 \\ x \neq 1 \end{matrix}$$

$$\frac{\frac{1}{a-1}+1}{\frac{1}{a+1}-1} = \frac{1+(a-1)}{a-1} : \frac{1-(a+1)}{a+1} = \frac{1+a-1}{a-1} \cdot \frac{a+1}{1-a-1} = \frac{a+1}{a-1} \cdot \frac{a+1}{-a} = -\frac{a+1}{a-1} \quad \begin{matrix} a \neq 1 \\ a \neq -1 \\ a \neq 0 \end{matrix}$$

$$\frac{\frac{b-a}{2a-b}}{4a^2-4ab+b^2} = \frac{b-a}{2a-b} : \frac{(a-b)(a+b)}{(2a-b)^2} = \frac{(-1)(a-b)}{2a-b} \cdot \frac{(2a-b)}{(a-b)(a+b)} = -\frac{2a-b}{a+b}$$

$$\begin{matrix} b \neq 2a \\ a \neq b \\ a \neq -b \end{matrix}$$

2. Upravte, určete podmínky platnosti.

$x \neq 1$   
 $x \neq -1$

$$\frac{\frac{4x^2-4}{x-1}}{(2x+2)^2} = \frac{(2x-2)(2x+2)}{x-1} : \frac{(2x+2)^2}{(x-1)(x+1)} = \frac{(2x-2)\cancel{(2x+2)}}{\cancel{x-1}} \cdot \frac{\cancel{(x+1)}}{(2x+2)^2} = \frac{(2x-2)(x+1)}{2x+2} = \frac{\cancel{2(x-1)}(x+1)}{\cancel{2}(x+1)} = \underline{x-1}$$

$$\frac{\frac{2p}{q}}{\frac{p^2q^2}{2q}} = \frac{2p}{q} : \frac{p^2q^2}{2q} = \frac{2\cancel{p}}{\cancel{q}} \cdot \frac{2\cancel{q}}{p^2q^2} = \frac{4}{p^2q} \quad \begin{matrix} p \neq 0 \\ q \neq 0 \end{matrix}$$

$$\frac{\frac{1+\frac{1}{s}}{t-\frac{s}{s}}}{\frac{t-s}{s}} = \frac{s+t}{st} : \frac{t^2-s^2}{st} = \frac{\cancel{s+t}}{\cancel{st}} \cdot \frac{\cancel{st}}{(t-s)(t+s)} = \frac{1}{t-s} \quad \begin{matrix} t \neq s & t \neq 0 \\ t \neq -s & s \neq 0 \end{matrix}$$

$$\frac{\left(\frac{2a^2}{b}\right)^3}{\frac{(4a)^2}{b^4}} = \frac{8a^6}{b^3} : \frac{16a^2}{b^4} = \frac{\cancel{8}a^{\cancel{6}}}{\cancel{b^3}} \cdot \frac{\cancel{16}b^4}{\cancel{2}^2} = \frac{a^4b}{2} \quad \begin{matrix} a \neq 0 \\ b \neq 0 \end{matrix}$$

$$\frac{1+\frac{n}{m}}{n-\frac{m^2}{n}} = \frac{m+n}{m} : \frac{m^2-m^2}{m} = \frac{\cancel{m+n}}{m} \cdot \frac{m}{(m-m)(\cancel{m+n})} = \frac{m}{m(m-m)} \quad \begin{matrix} m \neq m \\ m \neq -m \\ m \neq 0 \\ m \neq 0 \end{matrix}$$

$$\frac{\frac{(-3a^2)^3}{6ab}}{\frac{(-2ab)^2}{3a^2b}} = \frac{-27a^6}{6ab} : \frac{4a^2b^2}{3a^2b} = \frac{-27a^{\cancel{6}}}{\cancel{6}ab} \cdot \frac{\cancel{3}a^{\cancel{2}}b}{4a^2b^2} = \frac{-27a^5}{8b^2} \quad \begin{matrix} a \neq 0 \\ b \neq 0 \end{matrix}$$

$$\frac{4-\frac{y^2}{x^2}}{6-\frac{3y}{x}} = \frac{4x^2-y^2}{x^2} : \frac{6x-3y}{x} = \frac{(2x-y)(2x+y)}{x^2} \cdot \frac{x}{3(2x-y)} =$$

$$\frac{\frac{a}{2b} + \frac{b}{a}}{\frac{-a^2}{b^2} - \frac{1}{a}} = \frac{a}{2b} : \left(-\frac{a^2}{b^2}\right) + \frac{b}{a} : \left(-\frac{1}{a}\right) = \frac{\cancel{a}}{2\cancel{b}} \cdot \left(\frac{\cancel{b^2}}{a^2}\right) + \frac{\cancel{b}}{\cancel{a}} \cdot \left(-\frac{\cancel{a}}{1}\right) = -\frac{1}{2a} + (-b) = -\frac{1}{2a} - b = \frac{-1-2ab}{2a} \quad \begin{matrix} a \neq 0 \\ b \neq 0 \end{matrix}$$

$$\frac{-1-\frac{x^2}{y}}{x^4-y^2} = \frac{-y-x^2}{y} : \frac{(x^2-y)(x^2+y)}{y^2} = \frac{\cancel{-(y+x^2)}}{y^2} \cdot \frac{y}{(x^2-y)\cancel{(x^2+y)}} = \frac{-y}{x^2-y} \quad \begin{matrix} y \neq 0 \\ y+x^2 \\ y-x^2 \end{matrix}$$